

# Spontaneous $SU(2)$ symmetry violation in the $SU(2)_L \times SU(2)_R \times SU(4)$ electroweak model

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## Abstract

A new approach to EW composite scalars is developed, starting from the fundamental gauge interaction on high scale. The latter is assumed to have the group structure  $SU(2)_L \times SU(2)_R \times SU(4)$  where  $SU(4)$  is the Pati-Salam color-lepton group. The topological EW vacuum filled by instantons is explicitly constructed and the resulting equations for fermion masses exhibit spontaneous  $SU(2)$  flavor symmetry violation with possibility of very large mass ratios.

## 1 Introduction

It is a rather common assumption, that the Electroweak Theory (EW) in its standard version is an effective low-energy theory, produced by a fundamental interaction at a high scale. If this fundamental theory exists, it should answer many important questions, unanswered by the EW theory in its standard form, for example:

1. What is the dynamical origin of the Higgs field phenomenon and how the Electroweak Symmetry Breaking (EWSB), proceeds in fundamental terms from  $SU(2)_W \times U(1)_Y$  to  $SU(2)_W \times U(1)_{em}$ , yielding masses of  $W$  and  $Z$ .
2. What is the dynamical mechanism producing fermion masses.
3. What is the dynamical origin of generations.

4. How one can explain the pattern of fermion masses and mixings, in one generation and in particular, a large difference between masses of quarks and neutrinos.
5. What is the origin of the CP violation in EW.

More questions can be added to that list, e.g. why at all the  $SU(2)_L \times U(1)$  structure appears, and then why it is broken by unequal masses of  $t$  and  $b$  quarks and why left-right symmetry is broken. On the other hand, many physicists worked in the field last 50 years, producing a well-established and an accurately checked picture of EW theory in good agreement with experiment [1]. Numerous efforts have been done to answer the first question, suggesting dynamical models of composite Higgs mechanism. In a simpler form it was suggested to explain the composite Higgs field via the top condensation mechanism (see [2] for a review and references); in another version the Technicolor model (TC) was suggested (see [3] and [4] for reviews and references). In the extended versions (Extended Technicolor (ETC), Walking Technicolor (WTC)) the Lagrangian contains both usual color, TC and flavor interaction in one ETC gauge group [5], for a recent review see [6].

The common to all these approaches (actually trying to answer the points 1 and 2 above) is that a quark (plus possibly technoquark) condensate is formed at a high scale, which gives mass to quarks and the same mechanism then provides EWSB, yielding masses to  $W$  and  $Z$ .

As a consequence of TC and ETC models, additional particles in the region of 1 TeV and higher was predicted, which can possibly be detected experimentally.

To our knowledge no dynamical explanation for so different fermion masses and the structure of generations was given up to now in this type of studies. Recently a new general approach was suggested in [7], with a tentative explanation of dynamical origin of generations and the hierarchy of fermion masses. In particular, a simple pattern of masses and mixings, called the Coherent Mixing (CM) was developed for 3 and 4 generations and successfully compared to experimental data in [8].

However, the group structure of the dynamical mechanism was not discussed in [7, 8], neither the topic of EWSB was elaborated there, therefore we undertake below these two tasks and plan to discuss in detail the change of group structure with the energy scale and the form of the corresponding effective Lagrangians. We assume below, that at some high scale only exist-

ing fermions are participating and new gauge interactions can appear, which are frozen at lower scales.

We start with one family –more families, as shown in [7] can be produced as additional solutions of the same dynamical equation.

Each family of fermions consists of 16 members  $f_a^\alpha \equiv (u_L^\alpha, d_L^\alpha, d_R^\alpha, u_R^\alpha)$ ,  $\alpha = 1, 2, 3$  (color),  $\alpha = 4$ – leptons, so that  $f_a^4 = (\nu_L, e_L, \nu_R, e_R)$ . It is evident that indices  $\alpha, a$  can be organized into  $U(4)_a \times U(4)_\alpha$  or  $SU(4)_a \times SU(4)_\alpha$  group indices. The  $SU(4)_\alpha$  was introduced earlier in [9] as a unification of  $SU(3)_{color}$  and lepton number. It is clear, that  $SU(4)_\alpha$  is splitted down to  $SU(3)_{color} \equiv SU(3)_c$  at high scale, but we also know, that  $SU(2)_W$  is symmertic to quarks and leptons, and this calls for considering the possibility of  $U(4)_\alpha$  or  $SU(4)_\alpha$  symmetry. We shall have in mind the splitting pattern

$$SU(4)_\alpha \rightarrow SU(3)_c \times U(1)_{B-L} \quad (1)$$

Concerning the  $a$ -indices, we shall consider the flavor group  $SU(4)_a \equiv SU(4)_{EW}$ . However,  $SU(4)_a$  for a vector fundamental interaction automatically breaks down to  $SU(2)_L \times SU(2)_R$ , and we shall study explicitly the latter group, so that one essentially uses the group  $G(2, 2, 4) \equiv SU(2)_L \times SU(2)_R \times SU(4)$ , introduced and exploited in [9],[10]. The splitting pattern for that is similar to the standard one

$$E_6 \text{ or } SO(10) \rightarrow G(2, 2, 4) \rightarrow SU(2)_W \times U(1)_Y \rightarrow SU(2)_W \times U(1)_{em}. \quad (2)$$

In  $G(2, 2, 4)$  each fermion  $\psi_{a\alpha}$  can have gauge interaction of three types:  $A_\mu^{ab}, C_\mu^{a\alpha, b\beta}, B_\mu^{\alpha\beta}$ .

The  $B_\mu^{\alpha\beta}$  coincides with the usual color interaction when  $SU(4)_\alpha$  splits into  $SU(3)_c$ . The first,  $A_\mu^{ab}$  is the intrinsic EW interaction at a high scale producing, after the first splitting in (2), the gauge fields  $W_\mu^A$  and  $B_\mu$ , and after EWSB, the fields  $W_\mu^A, A = 1, 2$  and electromagnetic  $U(1)$  field. Now the field  $C_\mu^{a\alpha, b\beta}$  is local gauge field, which is adjoint both in  $a, b$  and in  $\alpha, \beta$  indices, and the latter will be considered both in splitted and unsplitted forms. We assume this interaction to be active at high scale  $M$  in an unbroken form, and surviving at low scale in the form of vacuum correlators, producing nonzero average fermion bilinears and hence effective composite scalars, which give masses to fermions, and in the course of EWSB also to vector fields.

We note at this point the possibility of inclusion into the game new particles, like technofermions, which could interact also with the field  $C_\mu^{a\alpha, b\beta}$  and

forming in this way the condensate of technofermions, but this line will not be pursued further.

It is clear, that the original  $G(2, 2, 4)$  group is badly broken, and therefore this should be reflected in the structure of vacuum averages of the fields  $\hat{A}_\mu, \hat{C}_\mu$ .

In this way the properties of the vacuum fields and vacuum symmetries are entering our problem together with the problem of the explicit mechanism of symmetry violation. The related problem is that of gauge invariance, since all fields  $\hat{A}_\mu, \hat{B}_\mu, \hat{C}_\mu$  are local gauge fields. As a general statement, we shall assume always the mechanism of mass generation and symmetry breaking to be associated with vacuum field correlators, which can generate scalar parts in the deconfined phase due to specific vacuum fields. An example of such fields is given by instantons, which can produce masses and chiral symmetry breaking (CSB), but do not give confinement at large distances. This latter fact is due to exact mutual cancellation of all correlators of higher powers in fields of this topological class at large distances [11], hence no area law for the Wilson loop, but finite scalar contributions in the Green's functions of quarks at smaller distances, producing in this way masses of quarks. Note, that this mechanism of CSB, studied before in [12], is different from the one, generated by confinement of QCD [13].

As we shall see below, the quadratic correlator of the field  $C_\mu^{a\alpha, b\beta}$  can produce quartic combination of quark fields, both of vector-vector and scalar-scalar kind. The latter combination is exactly what is considered in topcondensate or TC models, however, as will be seen, to produce this combination of two **white** bilinears (which is necessary for fermion masses), one needs to start with the field  $C_\mu^{a\alpha, b\beta}$ , depending on color indices  $\alpha, \beta$  as an adjoint operator. Since bilinears depend also on  $a, b$ , the same field  $\hat{C}_\mu$  should also contain these indices and be adjoint  $SU(2)_{EW}$  field before symmetry breaking. Another important outcome of the same mechanism, composite vector fields appear on the same ground as the composite scalars, and add to the possible intrinsic vector fields  $\hat{A}_\mu$ . Composite vector fields have a long history [14] (for a good discussion and references see [15]), and have also been considered as candidates for EW gauge vectors [15][16]. This important topic needs a separate paper and will be considered elsewhere.

The paper is organized as follows. In the next section the topological structure of the EW vacuum is discussed and instanton ensembles are considered as concrete examples. The form of effective quartic quark Lagrangian

is derived in section 3 together with the effective Lagrangian of composite vector gauge fields. Spontaneous  $SU(2)$  flavor symmetry breaking is discussed in section 4. In section 5 the instanton-induced equation for fermion masses is derived. In section 6 the obtained results are summarized. Four appendices contain details of derivations used in the paper.

## 2 Topological vacuum of electroweak theory

As was discussed above, the central element of our construction (as well as in topcondensate model [2]) is the four-quark (and multiquark) effective Lagrangian, which is produced by the field correlators of the fundamental gauge fields. To have those field correlators one needs nontrivial nonperturbative structure of the vacuum, which should establish the following properties of resulting physical amplitudes:

1. Local gauge invariance of the scalar self-energy of fermions, which ensures the physical fermion mass.
2. Symmetry breaking of the EW group yielding finally  $SU(2)_W \times U(1)_{em}$  with partially violated  $SU(2)_W$ , while  $SU(3)_c$  and  $B, L$  are not violated.
3. Mass generation at low scale for all gauge fields involved, except  $SU(3)_c$ , where confinement is operating.
4. CP violation.

In the first item one needs a gauge invariant Chiral Symmetry Breaking (CSB) phenomenon for an isolated fermion due to vacuum fields. It is known, that CSB occurs due to scalar confinement and disappears in the QCD vacuum together with it at  $T \gtrsim T_c$  [13, 17]. It is also known, that CSB may occur in the instantonic model of the QCD vacuum [12],[18],[19]. As was shown in [13], the difference between these two cases of CSB can be formulated in terms of the Field Correlator Method (FCM) [20], where the fermion mass operator is represented as an integral of a sum over all connected correlators  $\ll F(1)...F(n) \gg$ ,  $M = \sum_n M_n$ , (see Appendix 1 for details). To establish gauge invariance, one starts with the gauge invariant operator of the fermion mass in the static field of heavy antifermion at some fixed point  $\mathbf{R}_0$ . It was shown in [13], that every connected correlator ensures

the linear term in the mass operator  $M_n \sim c_n |\mathbf{x} - \mathbf{R}_0| + c_n^{(1)}$ , where  $c_n, c_n^{(1)}$  are some constants.

It is crucial, what will be the result of summation over  $n$

$$M = |\mathbf{x} - \mathbf{R}_0| \sum c_n + \sum c_n^{(1)}. \quad (3)$$

In the QCD vacuum it is known, that the dominant contribution to the sum (3) comes from the lowest term with  $n = 2$  [11] and one obtains scalar confinement, which by itself implies CSB, and confinement creates “constituent mass”, which actually is the average energy of the confined quark [21]. In case of instantonic vacuum, all terms in the sum over  $n$  in (3) are important; moreover, as shown in [11], the sum  $\sum_n c_n$  vanishes for an ensemble of topcharges with integer fluxes, e.g. for instantons, and one obtains the finite value of  $M^{(1)} = \sum c_n^{(1)}$ , not depending on  $\mathbf{R}_0$ , and hence fully gauge invariant. This is the case of CSB without confinement.

Another situation occurs in QCD for  $T \gtrsim T_c$ , where the surviving nonconfining correlator  $D_1$  produces the vector part of the quark selfenergy operator, and it appears in the form of the real part of the Polyakov loop [22] from the same gauge invariant construction as discussed above, while confining correlators vanish at  $T \geq T_c$ .

Thus the requirement in the point 1 leads us to consider the EW vacuum as ensemble of instantons (or more general solutions with integer fluxes) for the group  $SU(2) \times SU(4)$ , or more general subgroups of  $E_6$  or  $SO(10)$ .

Now the very structure of  $SU(2)$  instanton solutions can help to establish the symmetry breaking. Namely, in  $4d$  gauge theory the basic element is the  $SU(2)$  instanton of Belavin et al. [23], which can be embedded in the  $SU(2) \times SU(4)$  construction in different ways.

It is remarkable, that the resulting vacuum averages do not violate  $SU(2) \times SU(4)$ , and the final phenomenon of spontaneous  $SU(2)_{EW}$  violation will occur spontaneously due to nonsymmetric fermion mass creation.

Finally, to establish CP violation, one must require, that the total density of topcharges should be nonzero. For simplicity, one can assume, that vacuum ensemble consists of only instantons (or antiinstantons), thus vacuum condensate of gauge fields explicitly violates CP (in addition this vacuum is more stable, than instanton-antiinstanton vacuum). Then, using ABJ anomaly relation, one can absorb the CP violating effect in the fermion phases of (almost) massless fermions of the first generation. In what follows, however, we shall concentrate on the topic of fermion mass generation, leaving the subject of CP violation for another publication.

We consider the  $SU(2)$  instanton field in singular gauge with global color orientation  $\Omega$

$$A_\mu(x) = \bar{\eta}_{a\mu\nu} \frac{(x-R)_\nu \rho^2 \Omega^+ \tau_a \Omega}{(x-R)^2[(x-R)^2 + \rho^2]}. \quad (4)$$

Averaging in the instanton ensemble over each instanton is assumed with the weight  $D\gamma = \prod_{i=1}^N d\Omega_i \frac{dR_i}{V_4}$ , and the following equation for averaging in  $d\Omega$  will be used [24] ( $N_c = 4$  for  $SU(4)$ )

$$\int d\Omega \Omega_{ab}^+ \Omega_{cd} = \frac{1}{N_c} \delta_{ad} \delta_{bc}, \quad (5)$$

and (here  $a, \alpha$  refer to the same group indices)

$$\begin{aligned} \int d\Omega \Omega_{a\alpha}^+ \Omega_{\beta b} \Omega_{a'\alpha'}^+ \Omega_{\beta'b'} = & \frac{1}{N_c^2 - 1} (\delta_{ab} \delta_{a'b'} \delta_{\alpha\beta} \delta_{\alpha'\beta'} + \delta_{ab'} \delta_{a'b} \delta_{\alpha\beta'} \delta_{\alpha'\beta}) - \\ & - \frac{1}{N_c(N_c^2 - 1)} (\delta_{ab} \delta_{a'b'} \delta_{\alpha\beta'} \delta_{\alpha'\beta} + \delta_{ab'} \delta_{a'b} \delta_{\alpha\beta} \delta_{\alpha'\beta'}). \end{aligned} \quad (6)$$

Hence for the averaging of the square of instanton field of one can readily deduce from (6)

$$\langle k^2 \rangle \equiv \int (\Omega^+ \tau^A \Omega)_{ab} (\Omega^+ \tau^B \Omega)_{a'b'} d\Omega = \frac{\text{tr}(\tau^A \tau^B)}{N_c^2 - 1} (\delta_{ab'} \delta_{a'b} - \frac{1}{N_c} \delta_{ab} \delta_{a'b'}) \quad (7)$$

This can also be rewritten for  $N_c = 2$  as (since  $\text{tr}(\tau^A \tau^B) = 2\delta_{AB}$ )

$$\langle k^2 \rangle = 4/3 t_{ab}^C t_{a'b'}^C \delta_{AB}, \quad t^C = \frac{1}{2} \tau^C. \quad (8)$$

This can be immediately applied to the  $SU(2)_{EW}$  field  $A_\mu^{ab}$ , if one consider the ensemble of  $SU(2)$  instantons in the  $SU(2)_{EW}$  vacuum,  $(A_\mu)_{ab} = \varphi_\mu^A (\Omega^+ \tau^A \Omega)_{ab}$  and averages over color orientations the partition function (see Appendix 2 for details of derivation)

$$Z = \int D\gamma D\psi D\bar{\psi} e^i \int \bar{\psi}(\hat{\partial}+m)\psi d^4x + \int \bar{\psi} \hat{A} \psi d^4x \quad (9)$$

$$\langle e^{\int \bar{\psi} \hat{A} \psi d^4x} \rangle_\Omega = e^{\sum_{i=1}^N \sum_{n=1}^\infty \frac{1}{n!} \langle \theta_i^n \rangle}, \quad (10)$$

where

$$\theta_i \equiv \int \bar{\psi} \hat{A} \psi d^4x \equiv \int \bar{\psi}_{a\alpha} (\hat{A})_{ab} \psi_{b\alpha} d^4x, \quad (11)$$

and  $\alpha$  are  $SU(4)_{lc}$  indices, not participating in the averaging procedure. Applying (7), (8) to  $\ll \theta_i^2 \gg$  and omitting the instanton index  $i = 1, \dots, N$ , one has

$$\ll \theta^2 \gg = 2 \int d^4x d^4y \langle \varphi_\mu^A \varphi_\mu^A \rangle_R (\bar{\psi}_{a\alpha}(x) \gamma_\mu t_{ab}^C \psi_{b\alpha}(x)) (\bar{\psi}_{a'\beta}(y) \gamma_{\mu'} t_{a'b'}^C \psi_{b'\beta}(y)). \quad (12)$$

One can see in (12) the square of the effective vector field of the  $W_\mu$ -type, which means, that  $SU(2)$  instantons produce effective vector fields with the unbroken symmetry. It is important, that color symmetry and  $SU(4)_{lc}$  are not broken, and the effective  $W_\mu$  field is color blind. However at this stage the effective scalars do not appear and we must use instantons in the field  $C_\mu^{a\alpha, b\beta}$  to produce those.

We start with the averaging over fields  $C_\mu^{a\alpha, b\beta}$  in the quadratic effective Lagrangian, similar to  $\ll \theta^2 \gg$  (12), but now in the ensemble of “double instantons” in  $SU(2) \times SU(4)$  group, which is proportional to

$$T \equiv \langle C_\mu^{a\alpha, b\beta} C_\nu^{a'\alpha', b'\beta'} \rangle_C \sim \langle (\Omega^+ t^A \Omega)_{ab} (\Omega^+ t^B \Omega)_{a'b'} (\omega^+ \tau^D \omega)_{\alpha\beta} (\omega^+ \tau^E \omega)_{\alpha'\beta'} \rangle_{\Omega, \omega} \quad (13)$$

Using (7), (8), one obtains the following general structure

$$T = \delta_{\mu\nu} \{ \mathcal{M}_1 \delta_{\alpha\beta} \delta_{\alpha'\beta'} \delta_{ab} \delta_{a'b'} + \mathcal{M}_2 \delta_{\alpha\beta} \delta_{\alpha'\beta'} \delta_{ab'} \delta_{a'b} + \mathcal{M}_3 \delta_{\alpha\beta'} \delta_{\alpha'\beta} \delta_{ab} \delta_{a'b'} + \mathcal{M}_4 \delta_{\alpha\beta'} \delta_{\alpha'\beta} \delta_{ab'} \delta_{a'b} \}, \quad (14)$$

which should multiply the product of bilinears  $S \equiv (\bar{\psi}_{a\alpha} \gamma_\mu \psi_{b\beta}) (\bar{\psi}_{a'\alpha'} \gamma_{\mu'} \psi_{b'\beta'})$ . Here  $\mathcal{M}_i = \mathcal{M}_i(x, y)$ . For the product  $TS$  one has to do in the terms proportional to  $\mathcal{M}_3, \mathcal{M}_4$  the Fierz transformation to avoid colored bilinears,

$$TS = J_\mu J_\mu \left( \mathcal{M}_1 + \frac{1}{2} \mathcal{M}_2 \right) + 2 \mathcal{M}_2 J_\mu^A J_\mu^A + 2 \mathcal{M}_3 \sum_i c_i (\bar{\psi} O_i t^A \psi) (\bar{\psi} O_i t^A \psi) + \left( \frac{1}{2} \mathcal{M}_3 + \mathcal{M}_4 \right) \sum_i c_i (\bar{\psi} O_i \psi) (\bar{\psi} O_i \psi). \quad (15)$$

Here  $J_\mu = \bar{\psi} \gamma_\mu \psi$ ,  $J_{\mu A} = \bar{\psi} \gamma_\mu t^A \psi$  and  $c_i = (-1, \frac{1}{2}, \frac{1}{2}, 1)$  for  $S, V, A, P$  variants;  $O_i = (1, \gamma_\mu, \gamma_\mu \gamma_5, \gamma_5)$ .



Note, that  $a, b$  in (15) run over a pair of  $SU(2)$  indices and Dirac  $\frac{1 \pm \gamma_5}{2}$  structure defines whether they belong to  $SU(2)_L$  or  $SU(2)_R$ . For the scalar current part Eq. (15) one has

$$(TS)_{\text{scal}} = \sum_{i,k} \Lambda_{ik} \varphi_i \varphi_k^+ = -4 \left\{ (\mathcal{M}_3 + \mathcal{M}_4) (\varphi_1 \varphi_1^+ + \varphi_2 \varphi_2^+ + \varphi_1 \varphi_2^+ + \varphi_2 \varphi_1^+) + \right. \\ \left. + \mathcal{M}_3 (-\varphi_1 \varphi_2^+ - \varphi_2 \varphi_1^+ + \varphi_3 \varphi_3^+ + \varphi_4 \varphi_4^+) \right\} \quad (16)$$

where we have defined

$$\varphi_1 = \bar{u}_R u_L, \quad \varphi_2 = \bar{d}_R d_L, \quad \varphi_3 = \bar{d}_R u_L, \quad \varphi_4 = \bar{u}_R d_L, \quad \varphi_1^+ = \bar{u}_L u_R, \quad \text{etc.}$$

Finally,  $\mathcal{M}(x, y)$  are proportional to the trace of  $(C_\mu^{a\alpha, b\beta}(x) C_\mu^{a\alpha, b\beta}(y))$  averaged over positions and sizes of pseudoparticles, see Appendix 2 for details.

### 3 Study of quartic quark operators

As a result the partition function can be written as

$$Z = \int D\psi D\bar{\psi} e^{i \int \bar{\psi} \hat{\partial} \psi dx + \frac{1}{2} \int \langle (\bar{\psi} C \psi)^2 \rangle_c dx dy + \dots} \quad (17)$$

In this form the quartic in  $\psi, \bar{\psi}$  and higher terms appear, which shall be treated below.

As a next step, one can use the simple bozonization procedure to the vector terms in (16), introducing auxiliary vector field  $v_\mu^{(i)}(x, y), i = 1, 2$ . Using the identities

$$(\bar{\psi} J_i \psi)(\bar{\psi} J_i \psi) = (v^{(i)} - (\bar{\psi} J_i \psi))^2 - (v^{(i)})^2 + 2v^{(i)}(\bar{\psi} J_i \psi), \quad (18)$$

$i = \mu A, \quad \mu A 5, \mu, \mu 5, \quad J_{\mu 5} \equiv \bar{\psi} \gamma_\mu \gamma_5 \psi$ , one can write the partition function in the form

$$Z = \text{const} \int D\psi D\bar{\psi} Dv^{(i)} \exp \left\{ i \int \bar{\psi} \hat{\partial} \psi d^4x + \right. \\ \left. + \int dx dy \sum \xi_i(x, y) [-(v^{(i)})^2 + 2v^{(i)}(\bar{\psi} j_i \psi)] + (ST)_{\text{scal}}(\varphi, \varphi^+) \right\}. \quad (19)$$

Here  $\xi_i$  are expressed via  $\mathcal{M}_i$  in (14), the explicit form will not be used below.

With respect to the last term in (19) we apply the procedure, used in [7, 8], namely we introduce functional delta-function and write

$$e^{\int \Lambda_{ik} \varphi_i \varphi_k^+(x,y) d^4x d^4y} = \int \prod_{k=1}^4 D\mu_k D\mu_k^+ D\varphi_k D\varphi_k^+ e^{-K} \quad (20)$$

with

$$K = i \sum_{k=1}^4 \int [\mu_k(\varphi_k - (\bar{\psi} \Gamma_k \psi)) + \mu_k^+(\varphi_k^+ - (\bar{\psi} \bar{\Gamma}_k \psi))] dx dy + \int \sum_{ik} \Lambda_{ik} \varphi_i \varphi_k^+(x,y) dx dy \quad (21)$$

and  $\varphi_k, \varphi_k^+$  and  $\mu_k, \mu_k^+$  are to be found from the stationary points of the total effective Lagrangian (see [7] for more details). As a result due to (21) quark fields now enter only in bilinear forms in (20), and one can integrate over them yielding the final effective Lagrangian (note, that we also go over into the Minkowskian space-time (after all field averaging was done as always in the Euclidean space-time), and we have put  $\mu = \mu^+$  for simplicity, and the dash sign to show, that  $\mu$  is a matrix  $\mu_{ab}$ ).

$$L_{eff} = \sum \xi_i (v_\mu^{(i)})^2 - \frac{1}{2} \text{tr} \ln[(\hat{\mu} + i\hat{D})(\hat{\mu} - i\hat{D})], \quad (22)$$

where  $D_\mu$  contains now vector auxiliary fields

$$V_{1\mu}^{(A)} = 2\xi_1 v_\mu^{(1)A} + 2\xi_2 v_\mu^{(2)A} \gamma_5, \quad V_{2\mu} = 2\xi_3 v_\mu^{(3)} + 2\xi_4 v_\mu^{(4)} \gamma_5, \quad (23)$$

and can be written as

$$iD_\mu \equiv i\partial_\mu + V_{1\mu}^{(A)} t^A + V_{2\mu}. \quad (24)$$

To these terms in (23) one could add intrinsic EW fields  $g_2 W_\mu^a t^A + g_1 B_\mu \frac{Y}{2}$ , assuming that the field  $A_\mu^{ab}$  is broken to this  $(W, B)$  form by some other mechanism. We can account for this possibility, keeping in  $V_{1\mu}, V_{2\mu}$  the corresponding intrinsic parts.

The operator under the logarithm in (22) can be organized into several forms (writing matrix  $\hat{m}$  instead of  $\hat{\mu}$  in case  $\hat{\mu} \neq \hat{\mu}^+$ )

$$F \equiv (\hat{m} + i\hat{D})(\hat{m} - i\hat{D}) = \partial_\mu^2 + \hat{m}^2 + \Omega + \hat{N} \equiv d^2 + \hat{N} + \hat{\Omega}, \quad (25)$$

where

$$\hat{N} = \hat{V} \hat{m} - \hat{m} \hat{V}, \quad \hat{V} \equiv (V_{1\mu}^A t^A + V_{2\mu}) \gamma_\mu, \quad \hat{\Omega} = -i(\hat{\partial} \hat{V} + \hat{V} \hat{\partial}) - \hat{V}^2. \quad (26)$$

Hence the fermion loop expansion of  $tr \ln F$  can be written as

$$tr \ln F = tr \ln[d^2(1+G(\hat{N}+\hat{\Omega}))] = tr \ln d^2 + tr(G(\hat{N}+\hat{\Omega})) - \frac{1}{2}tr[G(\hat{N}+\hat{\Omega})G(\hat{N}+\hat{\Omega})] + \dots \quad (27)$$

where matrix  $G = (\hat{m}^2 + \partial^2)^{-1}$  and after diagonalization of  $\hat{m} \rightarrow \hat{m}_{\text{diag}}(a)$ ,  $G_a \equiv (d^2)_{aa}^{-1} = (\hat{m}_a^2 + \partial^2)_{aa}^{-1}$  is the free Green's function of fermion  $a = 1, 2$  with mass  $m_{\text{diag}}(a)$ , and higher terms of expansion are neglected.

From (22), (25) it is clear, that  $\hat{m}$  plays the role of the fermion mass matrix, indeed for  $\hat{\mu} \neq \hat{\mu}^+$  the mass operator  $\hat{m}$  in (25) is

$$\hat{m} \equiv \frac{\hat{\mu} + \hat{\mu}}{2} + \frac{\hat{\mu} - \hat{\mu}^+}{2}\gamma_5, \quad \hat{\mu} = \begin{pmatrix} \mu_1 & \mu_4 \\ \mu_3 & \mu_2 \end{pmatrix} \quad (28)$$

$$\hat{\mu}^+ = \begin{pmatrix} \mu_1^+ & \mu_3^+ \\ \mu_4^+ & \mu_2^+ \end{pmatrix}$$

and  $\mu_k, \mu_k^+$  can be obtained, as in [7] differentiating  $\Lambda_{ik}\varphi_i\varphi_k^+$  in  $\varphi_k, \varphi_k^+$  correspondingly, which yields

$$\mu_i(x) = \int \Lambda_{ik}(x, y)\varphi_k^+(y)dy; \quad \mu_k^+(x) = \int \Lambda_{ik}(x, y)\varphi_i(y)dy \quad (29)$$

where the  $\Lambda_{ik}(x, y)$  is easily obtained from (16),

$$\Lambda_{ik} = -4 \begin{pmatrix} a & a-b & 0 & 0 \\ a-b & a & 0 & 0 \\ 0 & 0 & -b & 0 \\ 0 & 0 & 0 & -b \end{pmatrix}, \quad (30)$$

and  $a \equiv \mathcal{M}_3 + \mathcal{M}_4$ ,  $b = \mathcal{M}_3$ . Both  $\varphi_k, \varphi_k^+$  are to be found from  $F$  (25) as

$$-i\varphi_k = \frac{1}{2} \frac{\delta}{\delta\mu_k} tr \ln F = \frac{1}{2} tr \left( G \frac{\delta\hat{m}^2}{\delta\mu_k} \right), \quad -i\varphi_k^+ = \frac{1}{2} tr \left( G \frac{\delta\hat{m}^2}{\delta\mu_k^+} \right). \quad (31)$$

Since both  $G$  and  $\hat{m}^2$  are matrices, the connection of  $\varphi_k$  and  $\mu_l$  takes the form in the momentum space (see Appendix 3 for a derivation)

$$-i\varphi \equiv d(p) = \frac{\mu^+(p)}{2(p^2 + \mu\mu^+)}, \quad -i\varphi^+ = d^+(p) = \frac{\mu(p)}{2(p^2 + \mu\mu^+)} \quad (32)$$

therefore the final form of equations for  $\hat{\mu}, \hat{\mu}^+$  is

$$\mu_i(p) = \int K_{ik}(p, p_1) \mu_k(p_1) d^4 p_1 + \dots \quad (33)$$

$$\mu_i^+(p) = \int \bar{K}_{ik}(p, p_1) \mu_k^+(p_1) d^4 p_1 + \dots$$

and e.g.  $K \sim \Lambda_{ik}$  the dots signify contribution of higher powers of  $\mu, \mu^+$ . Thus one obtains in general nondiagonal  $SU(2)$  matrices for  $\hat{\mu}, \hat{\mu}^+$  and  $\hat{V}$ ; moreover, since  $\hat{V}$  and  $\hat{m}$  contain  $\gamma_5$ , the interaction is different for left and right particles, so that the resulting form of (33) violates both  $SU(2)$  flavor and left-right symmetry, as will be discussed in the next section.

## 4 Electroweak symmetry breaking

There are two facts of EW symmetry breaking,

1)  $SU(2)_{EW} \times U(1)_Y \rightarrow SU(2)_{EW} \times U(1)_{em}$ , and 2) breaking of  $SU(2)_{EW}$  by unequal mass terms of up and down fermions. We shall demonstrate below, that both types of EWSB are present in the resulting  $L_{eff}$  (22). We start with the point 2), and remark, that the form (32) with nondiagonal matrices  $\hat{K}, \hat{K}^+$  implies, that eigenvalues  $\hat{\mu}, \hat{\mu}^+$  are not equal. Moreover, one may have asymmetric in  $\mu, \mu^+$  solutions, which automatically leads to the CP violation. To study fermion mass eigenvalues of the matrix  $\hat{m}$ , we first simplify to the case  $\hat{\mu} = \hat{\mu}^+$ , where  $\hat{m}$  has the form (28) in the  $(\bar{u}, \bar{d}) \times (u, d)$  basis, so that the eigenvalues of  $\hat{m} = \hat{\mu}$  are easily found to be

$$\bar{m}_{1,2} = \frac{\mu_1 + \mu_2}{2} \pm \sqrt{\left(\frac{\mu_1 - \mu_2}{2}\right)^2 - \mu_3 \mu_4}. \quad (34)$$

Now  $\mu_i$  are to be defined from (33), and one finds matrices  $K, \bar{K}$  etc. from (31). In the case  $\hat{\mu} = \hat{\mu}^+$  one derives a system of equations (see Appendix 3 for details of derivation)  $\mu_i = \int \Lambda_{ik} d(\mu_k)$  and we omit the integration signs,

$$\begin{aligned} \mu_1 &= -4[(\mathcal{M}_3 + \mathcal{M}_4)d(\mu_1) + \mathcal{M}_4 d(\mu_2)] \\ \mu_2 &= -4[(\mathcal{M}_3 + \mathcal{M}_4)d(\mu_2) + \mathcal{M}_4 d(\mu_1)] \\ \mu_3 &= -4\mathcal{M}_3 d(\mu_4) \\ \mu_4 &= -4\mathcal{M}_3 d(\mu_3), \end{aligned} \quad (35)$$

where  $d(\mu_i) = \frac{\mu_i}{p^2 + m^2}$ .

Hence the symmetry of the matrix  $\Lambda$  defines the symmetry of eigenvalues  $\mu_i$ . In particular, when  $\mu_3$  and /or  $\mu_4$  vanishes, one automatically obtains  $\mu_1 = \mu_2 = \bar{m}_{1,2}$  and no  $SU(2)$  flavor violation results. In the specific case, when  $a = 0$  in (30), one has  $\mu_1 = \mu_2 = \mu_3 = -\mu_4$ , and  $\bar{m}_1 = 2\mu_1, \bar{m}_2 = 0$ , exemplifying the maximal flavor violation. It is important to note, that by diagonalization of  $\hat{m}$  one defines the “true” up-and down fermions, which are rotated with respect to original  $u, d$  states.

Now we turn to the point 1).

It is useful to represent vector potential  $V_\mu$  (24) in the  $(L, R) \times$  (up, down) matrix notations, so that

$$\begin{aligned} V_\mu &= (V_L^A + V_R^A)t^A + V_L\hat{1} + V_R\hat{1} \equiv \bar{V}_L + \bar{V}_R = \\ &= \begin{pmatrix} V_L^A t^A + V_L\hat{1} & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & V_R^A t^A + V_R\hat{1} \end{pmatrix} \end{aligned} \quad (36)$$

and  $\hat{m} = \begin{pmatrix} 0 & \hat{\mu}^+ \\ \hat{\mu} & 0 \end{pmatrix}$ , so that (for  $\hat{\mu} = \hat{\mu}^+$ ) one obtains

$$\hat{N} = \hat{V}\hat{\mu} - \hat{\mu}\hat{V} = \begin{pmatrix} 0 & \bar{V}_L\hat{\mu} - \hat{\mu}\bar{V}_R \\ \bar{V}_R\hat{\mu} - \hat{\mu}\bar{V}_L & 0 \end{pmatrix}. \quad (37)$$

The basic role in the EWSB is played by the last term in (27), which can be rewritten as (here  $tr \equiv \frac{1}{4}tr_D tr_{a,b}$ , subscript  $D$  refers to Dirac matrices  $\gamma_\mu$ ).

$$tr \ln F = \dots - \frac{1}{2}[tr(G\hat{N}G\hat{N}) + tr(G\hat{\Omega}G\hat{\Omega})]. \quad (38)$$

As a first example we take for illustration the case, when  $\hat{V} = V_L^A t^A + V_2 y(a)$ , one obtains in the standard way [2, 25]

$$tr(G\hat{N}G\hat{N}) = -\frac{1}{2}(V_{1\mu}^A)^2(\mu_u^2 + \mu_d^2)G_u G_d - \frac{1}{2}[2(y_1 - y_4)V_{2\mu} + V_{1\mu}^3]^2(\mu_u^2 G_u^2 + \mu_d^2 G_d^2) \quad (39)$$

where we have introduced the hypercharge  $Y_a$  of the fermion  $a$ ,

$$y_{ab} = y(a) = diag(y_1, y_2, y_3, y_4), \quad y_a = \frac{Y_a}{2} \quad (40)$$

Here  $G^2$  is actually the integral cut-off at large scale  $M$

$$G_{u,d}^2 \rightarrow \frac{1}{(2\pi)^4} \int \frac{d^4 p}{(p^2 + \mu_{u,d}^2)^2} = \frac{1}{16\pi^2} \int_0^{M^2} \frac{p^2 dp^2}{(p^2 + \mu_{u,d}^2)^2}. \quad (41)$$

A more general case, EWSB, when both  $\hat{V}_L$  and  $\hat{V}_R$  are present in  $\hat{N}$  (37), is considered in Appendix 4. In this case additional terms appear in the resulting effective action making the whole picture of EWSB more complicated, which will be treated elsewhere [26].

## 5 Properties of selfconsistent solutions

In this section we analyze the properties of solutions for mass eigenvalues of Eq. (35), using Eq.(A2.16) for  $\mu(p)$ , tfor he kernels  $\bar{J}_n$ , obtained in Appendix 2 for the randomized ensemble of instantons. We also distinguish  $\mu$  and  $\mu^+$ , since (as also shown in Appendix 2) topological zero modes have definite chirality and produce different contribution to  $\mu$  and  $\mu^+$ . The resulting equations can be rewritten from (A2.16) as (all  $\mu, \mu^+, d, d^+$  are matrices in flavor space).

$$\begin{aligned} \mu(p) = & \int \frac{d^4 p_1}{(2\pi)^4} \bar{J}_2(p, p_1) d^+(p_1) - \int \prod_{i=1,2,3} \frac{d^4 p_i}{(2\pi)^4} \bar{J}_4(p, p_1, p_2, p_3) \times \\ & \times d^+(p_1) d(p_2) d^+(p_3) + \int \prod_{i=1}^5 \bar{J}_6(p, p_1, \dots, p_5) d^+(p_1) d(p_2) \dots d^+(p_5) + \dots \end{aligned} \quad (42)$$

For  $\mu^+(p)$  one should replace on the r.h.s.  $d^+ \leftrightarrow d$ . Here  $\bar{J}_n$  in the randomized instanton ensemble have the form (see Appendix 2 for details)

$$\bar{J}_2(p, p_1) = \frac{N_{top} k^2(q) q^2}{V_4 N_c^2 (2\pi)^4} = \left(\frac{\rho}{R}\right)^4 \frac{\chi^2(p - p_1)}{N_c^2 (2\pi)^4 (p - p_1)^2}, \quad (43)$$

where

$$\begin{aligned} \chi(q) = & \begin{cases} -\frac{1}{2}, & q \rightarrow 0 \\ -2/(q\rho)^2, & q \rightarrow \infty, \end{cases} \quad \text{and } k(q) = \frac{\chi(q)}{q^2} \\ \bar{J}_4(p, p_1, p_2, p_3) = & \frac{N_{top}}{4V_4} \left( \frac{4\rho^4}{(2\pi)^4} \right)^2 \prod_{i=1}^4 k(q_i) (q_1 q_2) (q_3 q_4'), \end{aligned} \quad (44)$$

and  $q_1 = p - p_1, q_2 = p_1 - p_2, q_3 = p_2 - p_3, q_4' = p - p_3, q_4 = -q_4'$ .

The Eq.(A4.10) allows to find solutions for  $\mu(p)$  and  $\mu^+(p)$ , since  $d(p)$  and  $d^+(p)$  are expressed via  $\mu, \mu^+$ : in the simple approximation  $\mu = \mu^+$  one has  $d(p) = d^+(p) = \frac{\mu(p)}{p^2 + \mu^2(p)}$  and in case of right zero mode contribution,  $d(p)$  and  $d^+(p)$  are given in (A2.29) and (A2.30) respectively.

As a next point, in this section we discuss the general structure of the spectrum of Eq. (42) using the random instanton vacuum (RIV) as an example. In this case, taking  $J_n(p, p_1, \dots, p_{n-1})$  from Appendix 2, Eqs.(A2.14), (A2.16) one can rewrite Eq. (42) via dimensionless combinations

$$\mu\rho = \frac{\rho^4}{4R^4} \int \prod_{i=1}^{n-1} \left( \frac{d^4 p_i (\mu(p_i)\rho)}{q_{i+1}^2 (p_i^2 + \mu^2(p_i))} \right) \prod_{k=1,3,\dots}^{n-1} \left[ \frac{4\rho^2 (q_k q_{k+1}) \chi_k \chi_{k+1}}{\sqrt{N_c}} \right] \quad (45)$$

Here  $\rho$  is the instanton size parameter,  $R^4 = V_4/N_{top}$  is the inverse instanton density,  $q_i = p_i - p_{i+1}$ ,  $p_{n+1} \equiv p_1$ . One can see, that  $\chi_k \cong -\frac{2}{(q_k \rho)^2}$ ,  $|q_k \rho| \rightarrow \infty$ , plays the role of the cut-off factor, and in the region of large  $p_i$  the integrand is well behaved, while in the infrared region ( $p_i \rightarrow 0$ ) the first term on the r.h.s. is logarithmically divergent for  $\mu \rightarrow 0$ , while all others are IR safe. Moreover, it is clear, that the n-th term is proportional to  $(\mu\rho)^{n-1}$ , and since  $\rho \sim 1/M$ , one expects, that  $\frac{\mu}{M} \ll 1$  for all roots of (45). Hence all terms with  $n > 2$  in this case of RIV should be subleading and can be omitted in the first approximation.

We shall concentrate now on the first term  $J_2$  in (42) and rewrite it for the RIV as (assuming at this stage  $\mu(p) = \mu^+(p)$  and omitting flavor matrix indices)

$$\mu(p) = b \int \frac{\mu(p_1) d^4 p_1}{(p - p_1)^2 (p_1^2 + \mu^2(p_1)) (2\pi)^4} \quad (46)$$

with  $b = \frac{\rho^4}{4N_c R^4}$ , and having in mind the cut-off at large  $p_1 \approx 1/\rho$  due to the suppression factor  $\chi^2(p - p_1)$ , not shown in (46). As a first guess one can put  $\mu(p) = \mu_0 = \text{const} \ll 1/\rho$ , and obtains ( $M \equiv 1/\rho$ )

$$1 \cong \frac{b}{16\pi^2} \ln \frac{M^2}{\mu_0^2}, \quad \mu_0^2 = M^2 \exp \left( -\frac{16\pi^2}{b} \right). \quad (47)$$

One can see, that for small  $\frac{b}{16\pi^2} \ll 1$  the resulting values of  $\mu_0$  can be very small, so that (47) illustrates the mechanism of small fermion mass due to RIV. Note, that the instanton solution was important in this, since it produces additional factor  $\frac{1}{(p-p_1)^2}$  in (46).

In appendix 2, one can see that the same type of factor appears for the case  $\mu \neq \mu^+$  due to zero mode contributions. Note, that the logarithmic instability of (46) is a typical feature, which is revealed in the 3d nonrelativistic potential  $1/r^2$ . One expects in this case, that there are additional

(excited) states, provided the strength of potential is large enough. It is important to study possible mechanisms of relaxation at large distances in RIV, which exist for the mixture of instanton and antiinstantons. However, for the homogeneous instanton gas without antiinstantons one can have a sum of individual contributions without interaction between instantons, and thus the relaxation may be absent.

Now we restore the flavor indices and consider Eq. (47) diagonalized in the flavor space, so that  $\mu(p) \rightarrow \mu_i(p)$ ,  $b \rightarrow b_i$ ,  $i = 1, 2$ . As can be seen in (46), the small and unqual  $b_1 \neq b_2$  can lead to very different  $\mu_0^{(i)}$ ,  $\mu_0^{(i)} = \mathcal{M} \exp\left(-\frac{8\pi^2}{b_i}\right)$  thus strongly amplifying the disparity of coefficients  $b_i$  defined by  $\Lambda_{ik}$  (the “zoom effect”).

## 6 Summary and conclusions

We have obtained above two basic results. First, we have derived equations for fermion masses  $\mu_k, \mu_k^+$ , Eq. (33), which defined selfconsistently both diagonalized (physical) fermion masses  $\mu_d$ ,  $\hat{\mu} = U\mu_d U^+$ ,  $U = a + i\mathbf{n}\boldsymbol{\tau}$ , and  $\mu_d^+$ . The equations contain the kernel  $\mathcal{M}(x, y)$ , which is proportional to the quadratic field average  $\langle C_\mu(x)C_\mu(y) \rangle$ , and can be explicitly calculated for the random instanton ensemble. As it is, the nondiagonal structure of kernels  $K_{ik}$  etc., already seen in (30), presupposes  $SU(2)$  violation in fermion masses, revealed in unequal masses of up and down fermions. Note, that our original interaction  $C_\mu^{a\alpha, b\beta}$  and its quadratic average do not violate  $SU(2)_L$  or  $SU(2)_R$ , and this violation occurs spontaneously in the final equations (35) resulting from the stationary points of the effective action (12). Another basic result of the present paper is the integral equation for fermion mass eigenvalues (46), where the kernel is provided by instanton density and shape, yielding approximate solutions of the form (47), highly sensitive to the small differences in the kernel values for two diagonal mass eigenvalues.

This latter equation (47) is able in principle provide large ratios of masses in the same  $SU(2)$  multiplet, as in  $m_t/m_b$ , and in ratios of quark to lepton masses. This type of analysis is planned for the future. A different scenario to explain large mass ratios and  $SU(2)$  flavor violation was suggested recently in [27], where a new type of horizontal interaction was introduced.

The results of the present study can be considered also in the framework of the left-right symmetric models [28, 29], with possible introduction of the effective composite scalars instead of elementary Higgs field. Another



useful connection could be with the recently proposed model [30], where left-right symmetric flavor symmetry is broken by the Yukawas. Finally, effective composite scalar of the kind considered in the present paper could be exploited in the GUT scenarios, based on  $SO(10)$  gauge symmetry [31, 32], where important quantum effects [33] can be formulated in terms of composite fields.

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## Appendix 1

### Chiral symmetry breaking and fermion mass generation

We start with the case of heavy quarks and consider the Green's function of a white pair of static quark and antiquark  $Q\bar{Q}$  at distance  $R$ , which is proportional to the Wilson loop  $(R \times T)$ , which can be written as a sum over the field correlators (cluster expansion) [11, 20]

$$\begin{aligned} W(S) &= \langle \exp(ig \int_S ds_{\mu\nu} F_{\mu\nu}(z)) \rangle = \\ &= \exp \sum_n \frac{(ig)^n}{n!} \int_S \dots \int_S ds_{\mu_1\nu_1}(u_1) ds_{\mu_2\nu_2}(u_2) \dots ds_{\mu_n\nu_n}(u_n) \times \\ &\quad \times D_{\mu_1\nu_1 \dots \mu_n\nu_n}(u_1, u_2, \dots u_n). \end{aligned} \quad (\text{A1.1})$$

We shall be interested only in the confining correlators, which can be written as

$$\langle \langle F_{\nu_1\mu_1}(u^{(i)}) \dots F_{\nu_n\mu_n}(u^{(n)}) \rangle \rangle = \prod_{i,k} (\delta_{\nu_i\nu_k} \delta_{\mu_i\mu_k} - \delta_{\nu_i\mu_k} \delta_{\nu_k\mu_i}) D^{(n)}(u^{(i)}, \dots u^{(n)}) \quad (\text{A1.2})$$

Separating the c.m. coordinates, yielding total area  $||S||$  of the surface  $S$ , one has for the flat minimal surface

$$W(S) = \exp(-||S|| \sum -(R+T)\kappa + O(R/T, T/R)), \quad (\text{A1.3})$$

$$\Sigma = \sum_{n=2k} \frac{(ig)^n}{n!} \int d^2u_1 \dots d^2u_n D^{(n)}(u_1, \dots, u_n)$$

where  $D^{(u)}$  depends only on relative coordinates  $(u_i - u_j)$ , which change in the intervals  $(-\infty, \infty)$ . Now for the field correlators of the vacuum configurations consisting of instantons with integer fluxes the sum  $\Sigma = 0$  in (A1.3), implying no confinement. However, there exist perimeter terms, (do not confuse these nonperturbative terms with diverging perturbative artefacts, specific for the Wilson loop of static quarks). These terms arise from the finite intervals of integration in  $\Sigma$  in (A1.3). E.g. for the quadratic correlator  $D^{(2)}$  one has the static potential

$$V_{Q\bar{Q}}(R) = 2 \int_0^R (R - \lambda) d\lambda \int_0^\infty d\nu D^{(2)}(\sqrt{\lambda^2 + \nu^2}) \quad (\text{A1.4})$$

and for large  $R$  one obtains

$$V_{Q\bar{Q}}(R \rightarrow \infty) = \sigma R + d_2, \quad (\text{A1.5})$$

with  $\sigma = 2 \int_0^\infty d\lambda \int_0^\infty d\nu D^{(2)}(\sqrt{\lambda^2 + \nu^2})$ ,  $d_2 = -2 \int_0^\infty \lambda d\lambda \int_0^\infty d\nu D^{(2)}(\sqrt{\lambda^2 + \nu^2})$ .

The sum over  $n$  of constant terms need not vanish, and one can write for the ensemble of integer fluxes

$$V_{Q\bar{Q}}(R \rightarrow \infty) = - \sum_{k=0}^\infty \frac{(-g^2)^k}{(2k)!} \int_0^\infty \lambda_1 d\lambda_1 \dots \lambda_{2k-1} d\lambda_{2k-1} d\nu_1 \dots d\nu_{2k-1} \times \quad (\text{A1.6})$$

$$D^{(2k)}(\lambda_1, \nu_1, \dots, \lambda_{2k-1}, \nu_{2k-1})$$

thus one obtains a finite scalar contribution to  $V_{Q\bar{Q}}(R)$  which does not depend on  $R$  at large  $R$  and one half of it can play the role of the effective mass of each of the static quarks, (which can be also negative). However, it was derived for static quarks, and we need equivalent expressions for fermions of any mass. To do that we shall exploit another technic, useful for the system of light fermion of any mass in the field of static antifermion.

Below we give the gauge invariant derivation of the fermion self-energy part in terms of field correlators, following the reference [13].

We start with the calculation of effective Lagrangian for the system of a colored fermion in the field of a static antifermion at point  $\mathbf{R}_0$ . It can be written in the leading order of the  $1/N_c$  expansion as follows:

$$\langle \exp \left[ \int \bar{\Psi}_\alpha \hat{B}^{\alpha\beta} \Psi_\beta d^4z \right] \bar{\psi}(x) P \exp(ig \int_y^x B_\mu(u) du_\mu) \psi(y) \rangle_{B, \psi, \bar{\psi}} \quad (\text{A1.7})$$

The averaging over  $B_\mu$  and  $\psi, \bar{\psi}$  yields the cumulant expansion in the exponent and brings about the following form of the self-energy part in the fermion-antifermion Green's function,

$$iM^{(n)}(x^{(1)}, \dots x^{(n)}) = \gamma_{\mu_1} S(x^{(1)}, x^{(2)}) \gamma_{\mu_2} \dots \gamma_{\mu_{n-1}} S(x^{(n-1)}, x^{(n)}) \gamma_{\mu_n} \times \\ \times N_{\mu_1 \dots \mu_n}^{(n)}(x^{(1)}, \dots x^{(n)}) \quad (\text{A1.8})$$

where we have defined

$$N_{\mu_1 \dots \mu_n}^{(n)} = \int_0^{x_1} d\xi_{\nu_1}^{(1)} \int_0^{x_2} d\xi_{\nu_2}^{(2)} \dots \int_0^{x_n} d\xi_{\nu_n}^{(n)} \alpha(\xi_{\nu_1}) \dots \alpha(\xi_{\nu_n}) \times \\ \ll F_{\nu_1 \mu_1}(\xi^{(1)}) \dots F_{\nu_n \mu_n}(\xi^{(n)}) \gg \quad (\text{A1.9})$$

and  $\alpha(\xi_4) = 1$ ,  $\alpha(\xi_i^{(k)}) = \frac{\xi_i^{(k)}}{x_i^{(k)}}$ ,  $i = 1, 2, 3; k = 1, \dots n$ . Here  $\xi^{(i)} = 0$  at  $\mathbf{R}_0$ .

One can identify in cumulant  $\ll \dots \gg$  the part similar to  $D$ , i.e. violating the Abelian Bianchi identity, namely for even  $n$

$$\langle \langle F_{\nu_1 \mu_1}(\xi^{(1)}) \dots F_{\nu_n \mu_n}(\xi^{(n)}) \rangle \rangle = \prod_{i,k} (\delta_{\nu_i \nu_k} \delta_{\mu_i \mu_k} - \delta_{\nu_i \mu_k} \delta_{\nu_k \mu_i}) D^{(n)}(\xi^{(1)}, \dots \xi^{(n)}) \quad (\text{A1.10})$$

Note, that the total Green's function of the fermion-static antifermion system is gauge invariant and for in and out coordinates  $x, y$  in (A1.7) can be written as

$$G(x, y, \mathbf{R}_0) = \text{tr}(\Phi(x, y, \mathbf{R}_0) S(x, y)) \quad (\text{A1.11})$$

where

$$\Phi(x, y, \mathbf{R}_0) = \Phi(\mathbf{x}, x_4; \mathbf{R}_0, x_4) \Phi(\mathbf{R}_0, x_4; \mathbf{R}_0, y_4) \Phi(\mathbf{R}_0, y_4; \mathbf{y}, y_4) \quad (\text{A1.12})$$

and  $\Phi$  is the parallel transporter

$$\Phi(a, b) = P \exp(ig \int_b^a B_\mu du_\mu) \quad (\text{A1.13})$$

Let us consider now the case of light quark  $q$  in the field of the static antiquark  $\bar{Q}$ . The initial and final states can be written in the gauge-invariant form as

$$\Psi_{in,out}(x, y) = \bar{\psi}_q(x) \Phi(x, y) \Psi_{\bar{Q}}(y). \quad (\text{A1.14})$$

The Green's function for the total  $q\bar{Q}$  system can be written as

$$G_{q\bar{Q}}(x, y | x', y') = \langle \Psi_{out}^+(x', y') \Psi_{in}(x, y) \rangle_{q,B} =$$

$$= \langle \text{tr}(\Phi(x', y') S_Q(y', y) \Phi(y, x) S_q(x, x')) \rangle_B, \quad (\text{A1.15})$$

and for the static quark one can take  $S'_q(y', y) \sim \Phi(y', y)$  where  $\Phi(y', y)$  is along the straight line, and for  $S_q$  one can use the Fock-Feynman-Schwinger Representation (FFSR) ,

$$S_q(x, y) = (m - \hat{D}) \int_0^\infty ds (Dz)_{xy} e^{-K} P_B \exp \left( ig \int_y^x B_\mu dz_\mu \right) p_\sigma(x, y; s) \quad (\text{A1.16})$$

Here  $\hat{D} = \hat{\partial} - ig\hat{B}$ ,  $P_B$  is the ordering operator, and  $p_\sigma$  is spin-dependent factor.

$$p_\sigma(x, y; s) = P_F \exp[g \int_0^s d\tau \sigma_{\mu\nu} F_{\mu\nu}(z(\tau))] \quad (\text{A1.17})$$

and  $\sigma_{\mu\nu} F_{\mu\nu}$  is

$$\sigma_{\mu\nu} F_{\mu\nu} = \begin{pmatrix} \boldsymbol{\sigma} \mathbf{H}, & \boldsymbol{\sigma} \mathbf{E} \\ \boldsymbol{\sigma} \mathbf{E}, & \boldsymbol{\sigma} \mathbf{H} \end{pmatrix} \quad (\text{A1.18})$$

As a result  $G_{qQ}$  can be written in a simpler form

$$G_{q\bar{Q}}(x, y|x', y') = \int_0^\infty ds (Dz)_{xx'} e^{-K} \langle \text{tr}(m - \hat{D}) W_\sigma(x, y|x', y') \rangle_B \quad (\text{A1.19})$$

and  $W_\sigma(x, y|x', y')$  is the Wilson loop with spin insertions of  $(\sigma F)$  factors and with contour  $C(x, y|x', y')$  consisting of the variable path from  $x$  to  $x'$  and three straight-line pieces  $\Phi(x', y')\Phi(y', y)\Phi(y, x) \equiv \Phi(x', y', y, x)$

$$W_\sigma(x, y|x', y') = \text{tr}[\exp(ig \int_{x'}^x B_\mu dz_\mu) p_\sigma(x, y; s) \Phi(x', y')\Phi(y', y)\Phi(y, x)] \quad (\text{A1.20})$$

Writing (A1.19) as a gauge invariant combination,

$$G_{q\bar{Q}} = \langle \text{tr} \Phi(x', y', y, x) S_q(x, x') \rangle_B \quad (\text{A1.21})$$

one can see, that we are interested in the situation, where there is Chiral Symmetry Breaking (CSB) so that  $\langle S_q(x, x) \rangle$  is nonzero, but confinement is missing, so that one can remove the heavy antiquark to infinity. To get this property, one starts with the ensemble of instantons, and keeping in the cluster expansion of the Wilson loop  $W_\gamma$  only the quadratic (Gaussian) term, one obtains confinement and CSB at the same time (see [13] for details). However, taking all correlators, one gets as in previous example of Wilson loop for static quarks, no linear term in the  $q\bar{Q}$  interaction – no confinement,

but may have remnants of scalar interaction, as in A1.4. This is enough for CSB, since appearance of scalar pieces in the effective interaction (effective mass) signals CSB. Thus one obtains CSB for the ensemble of instantons in absence of confinement. This phenomenon of CSB in instantonic vacuum was studied before in [12].

## Appendix 2

### Field correlators of a random instanton ensemble

We consider here the vacuum of SU(2) gauge theory filled by the noninteracting gas of instantons I (and possibly antiinstantons  $\bar{I}$ ), so that the total vector potential  $A_\mu$  is

$$A_\mu(x) = \sum_{i=1}^N A_\mu^{(i)}(x, \gamma_i) \quad (\text{A2.1})$$

where  $\gamma_i$  defines the set of collective coordinates: position, color orientation and size  $\gamma_i = \{R^{(i)}, \Omega_i, \rho^{(i)}\}$ , and we are interested in the effective fermion Lagrangian, which obtains after integration of partition function over all collective coordinates

$$Z = \int d\gamma D\psi D\bar{\psi} e^{\int \bar{\psi}(i\hat{\partial} + im + g\hat{A})\psi dx} = \int d\psi d\bar{\psi} e^{\int \bar{\psi}(i\hat{\partial} + im)\psi + L_{eff}(\psi, \bar{\psi})}. \quad (\text{A2.2})$$

Here notation is used

$$d\gamma = \prod_{i=1}^N \left( d\Omega_i \frac{d^4 R^{(i)}}{V_4} \right) \quad (\text{A2.3})$$

and the instanton potential in the singular gauge is

$$A_\mu^{(i)}(x, \gamma) = \bar{\eta}_{a\mu\nu} \frac{(x - R^{(i)})_\nu \rho^2 \Omega_i^+ \tau_a \Omega_i}{(x - R^{(i)})^2 [(x - R^{(i)})^2 + \rho^2]}. \quad (\text{A2.4})$$

It was shown in [19] that  $L_{eff}(\psi, \bar{\psi})$  can be found by the cluster expansion in the limit of large  $N_c$  (when SU(2) is the subgroup of  $SU(N_c)$ ) and when  $N, V_4 \rightarrow \infty$ ,  $\frac{N}{V_4} = \text{const}$  (the thermodynamical limit).

The result for  $L_{eff}$  is the sum over instantons ( $i$ ) and over power  $n$  of cumulant  $\ll (A_\mu)^n \gg$ , [19],

$$L_{eff} = \sum_{i=1}^N \sum_{n=2}^{\infty} \frac{1}{n!} \ll \theta_i^n \gg \equiv \sum_{i=1}^N L_{eff}^{(i)} \quad (\text{A2.5})$$

where we have denoted ( $f_i$ –flavor indices).

$$\theta_i = \sum_{f_i=1}^{N_f} \theta_i^{f_i}, \quad \theta_i^{f_i} \equiv \int dx \psi_{f_i}^+(x) \hat{S}^{(i)} \psi_{f_i}(x). \quad (\text{A2.6})$$

The calculation in [19] yields the following answer

$$L_{eff}^{(i)} = \sum_{n=2}^{\infty} \frac{N}{2nV_4N_c^2} \prod_{k=1}^n \int O_k \frac{dp_k dq_k}{(2\pi)^8} (2\pi)^4 \delta(\sum_1^n q_k) \text{tr} \{ \prod_{j=1}^n A_{\mu_j}(q_j) \} \quad (\text{A2.7})$$

where

$$O_k \equiv \psi_{\alpha_k}^+(p_k) \gamma_{\mu_k} \psi_{\beta_k}(p_k - q_k) \delta_{\alpha_k, \beta_{k-1}}. \quad (\text{A2.8})$$

For our purpose we need to form white bilinears ( $\psi_\alpha^+ \psi_\alpha$ ) using pairwise Fierz identities, keeping only scalar and pseudoscalar combinations and taking into account antisymmetry of  $\psi, \psi^+$ .

This yields

$$\prod_{k=1}^n O_k \rightarrow - \prod_{k=1,3,\dots}^n \Phi_{LR}^{(t_k)}(p_{k+1}, p_k - q_k) \Phi_{RL}^{(t_k)}(p_{k+2}, p_{k+1} - q_{k+1}) \quad (\text{A2.9})$$

where  $\Phi_{LR}$  denotes a fermion-antifermion pair,

$$\Phi_{LR}(p', p - q) = \psi_{Lf'}^+(p') \psi_{Rf}(p - q), \quad t = (f'f). \quad (\text{A2.10})$$

For the vacuum-averaged pair  $\langle \Phi_{LR} \rangle$  one has the property

$$\Phi_{LR}^{(t)}(p', p - q) = \delta(p' - (p - q)) \varphi_{LR}^{(t)}(p'). \quad (\text{A2.11})$$

As a result one can write  $L_{eff}^{(i)}$  in the form

$$L_{eff}^{(i)} = - \sum_{n=2}^{\infty} \frac{N_{top}}{2nN_c^2V_4} \prod_{i=1}^n \int \frac{d^4 p_i}{(2\pi)^4} T(p_1, p_2, \dots, p_n) \varphi_{LR}^{(t_1)}(p_1) \varphi_{RL}^{(t_1)}(p_2) \dots \varphi_{RL}^{(t_n)}(p_n) \quad (\text{A2.12})$$

with the notation

$$T(p_1, \dots, p_n) \equiv \prod_{i=1,3,\dots}^{n-1} \lambda(q_i, q_{i+1}); \lambda(q_i, q_{i+1}) = 4\rho^4 k(q_i)k(q_{i+1})(q_i q_{i+1}) \quad (\text{A2.13})$$

and  $q_i \equiv p_i - p_{i+1}$  with  $p_{n+1} \equiv p_1$ , and  $q_\mu k(q)$  is the Fourier transform of the instanton field,

$$k(q) \equiv \frac{1}{q^2} \left[ K_2(q\rho) - \frac{2}{(q\rho)^2} \right].$$

Comparing (A2.12) with eqs. (??), (18) from [7], one can define the correlators  $J_n$ ,

$$J_n(p_1, \dots, p_n) = \frac{N}{V_4 2n N_c^2} T(p_1, p_2, \dots, p_n). \quad (\text{A2.14})$$

Using the asymptotics of the modified Bessel function  $K_2(z)$  one has

$$q^2 k(q) \sim -\frac{1}{2} - O((q\rho)^2), \quad (q\rho) \rightarrow 0,$$

$$q^2 k(q) \sim -\frac{2}{(q\rho)^2} + \sqrt{\frac{\pi}{2q\rho}} e^{-q\rho} \left( 1 + O\left(\frac{1}{q\rho}\right) \right), \quad (q\rho) \rightarrow \infty. \quad (\text{A2.15})$$

Hence the equation for  $\mu(p)$ , eq. (18) of [7] acquires the form (with  $\varphi_{LR} = id^+, \varphi_{RL} = id$ )

$$\mu(p) = \frac{N}{4V_4 N_c^2} \sum_{n=2,4,\dots}^{\infty} \int \frac{d^4 p_1 \dots d^4 p_{n-1} (-)^{\frac{n}{2}}}{(2\pi)^{4(n-1)}} T(p, p_1, \dots, p_{n-1}) d^+(p_1) d(p_2) d^+(p_3) \dots d(p_{n-2}) d^+(p_{n-1}), \quad (\text{A2.16})$$

with  $d(p) \equiv \frac{\mu(p)}{p^2 + \mu^2(p)}$ ,  $d^+(p) = \frac{\mu^+(p)}{p^2 + (\mu^+(p))^2}$  and equation for  $\mu^+(p)$  is obtained by replacement  $\mu \leftrightarrow \mu^+, d \leftrightarrow d^+$ . The integral in (A2.16) converges both at small and large momenta, in the latter case symbolically as  $\frac{N}{2V_4 q^3} \left( \frac{\mu(p) d^4 p}{q^3 (p^2 + \mu^2(p))} \right)^{n-1}$ .

As a next step we consider the contribution of instanton zero modes to the quark propagator in the random instanton field, i.e. we consider as in [7] the total averaged field of randomized instantons  $C_\mu(x)$ , which produces higher field correlators  $J_n(x_1, \dots, x_n)$ , and in addition a number of topcharges (possibly also instantons), which produce zero modes. As in Eq. (26) of [7] one can write

$$C_\mu(x) \rightarrow \bar{C}_\mu(x) + \sum_{i=1}^N A_\mu^{(i)}(x - R_i). \quad (\text{A2.17})$$

Note, that in the randomized ensemble  $\bar{C}_\mu(x)$  we are not interested by the zero modes, while in the second term in (A2.17) only zero modes effects will be taken into account. Correspondingly, the fermion propagator assumes the form (cf. Eq. (A4.7) of [7], where difference in  $\mu, \mu^+$  was not taken into account)

$$S(p) = \frac{1}{\hat{p} - i\bar{\mu}(p)} + \frac{\bar{c}_0(p)l_+}{-i\mu_0}, \quad S^{-1} = \frac{(p^2 + \mu\mu^+)}{p^2 + \mu q}(\hat{p} - iql_- - i\mu l_+) \quad (\text{A2.18})$$

where

$$l_\pm = \frac{1 \pm \gamma_5}{2}, \quad q \equiv \mu + \frac{(p^2 + \mu\mu^+)\bar{c}_0}{\mu_0},$$

$$\bar{\mu}(p) = \frac{\mu(p) + \mu^+(p)}{2} + \gamma_5 \frac{\mu(p) - \mu^+(p)}{2} \quad (\text{A2.19})$$

and the term  $\mu(p)$  comes from  $\Phi_{LR}$ , while  $\mu^+(p)$  from  $\Phi_{RL}$ . Here  $\bar{c}_0(p)$  and  $\mu_0$  are obtained from zero modes of the ensemble of topcharges of the second term on the r.h.s. of (A2.17). For simplicity we shall consider instead one-instanton zero mode in the eigenvalue expansion ( $R$  is the position of instanton)

$$S(x, y) = \sum_n \frac{u_n(x - R)u_n^+(y - R)}{\lambda_n - i\mu}. \quad (\text{A2.20})$$

In case, when the effective fermion mass  $\mu = \mu(x, y)$  depends on coordinates, (A2.20) transforms into

$$S(x, y) = \sum_n \frac{u_n(x - R)u_n^+(y - R)}{\lambda_n \delta_{nm} - i\mu_{nm}}, \quad \mu_{nm} = \langle n | \mu(x, y) | m \rangle, \quad (\text{A2.21})$$

One can separate in (A2.21) the zero mode term and associate  $\mu_0$  in (A2.18) with  $\mu_{00}$

$$\mu_0 \equiv \mu_{00} = \int \mu(p)u_0^+(p)u_0(p) \frac{d^4p}{(2\pi)^4} \quad (\text{A2.22})$$

while nonzero modes are assembled in the first term in (A2.17). As was discussed in [7], the zero mode coefficient is proportional to  $|u_0(p)|^2$ , and here



we in addition to Eq. (41) of [7], also notice that one instanton produces a zero mode of definite chirality, i.e.

$$c_0(p) = \frac{N_0}{V_4} |u_0(p)|^2 (1 + \gamma_5) \equiv \bar{c}_0(p) \frac{1 + \gamma_5}{2} \quad (\text{A2.23})$$

where we generalized to the case of  $N_0$  instantons in the volume  $V_4$ .

We turn now to the zero mode wave function. The spacial part is given by

$$u_0(p) = \frac{\rho}{\pi} \int \frac{d^4 x e^{ipx}}{(\rho^2 + x^2)^{3/2}} = 2\rho \int_0^\infty \frac{J_0(pr) r^3 dr}{(\rho^2 + r^2)^{3/2}}. \quad (\text{A2.24})$$

The last integral can be expressed via generalized hypergeometric series

$${}_1F_2(a; b, c|z) = \left( 1 + \frac{a}{b \cdot c} \cdot \frac{z}{1!} + \frac{a(a+1)}{b(b+1)c(c+1)} \frac{z^2}{2!} + \dots \right)$$

$$u_0(p) = 2\rho \left\{ \frac{1}{p} {}_1F_2\left(\frac{3}{2}; \frac{1}{2}, \frac{1}{2}, \frac{p^2 p^2}{4}\right) - 2\rho {}_1F_2\left(2; \frac{3}{2}, 1; \frac{p^2 p^2}{4}\right) \right\}. \quad (\text{A2.25})$$

Expansion at small  $p$  is

$$u_0(p) = 2\rho \left\{ \frac{1}{p} - 2\rho + \frac{3}{2} p \rho^2 + O(p^2) \right\}. \quad (\text{A2.26})$$

Calculation of  $d(p)$  using (A2.18) with  $c_0(p)$  from (A2.23) yields

$$d(p) = \frac{\delta \text{tr} \ln S^{-1}(p)}{\delta \mu(p)} = \frac{\mu^+(p)}{2(p^2 + \mu(p)\mu^+(p))(1 + \frac{\bar{c}_0(p)}{\mu_0} \mu(p))} +$$

$$+ \frac{u_0^2(p)}{2\mu_0^2} \int \frac{d^4 p'}{(2\pi)^4} \frac{\mu(p') \bar{c}_0(p')}{(1 + \frac{\bar{c}_0(p')}{\mu_0} \mu(p'))} \quad (\text{A2.27})$$

$$d^+(p) = \frac{\delta \text{tr} \ln S^{-1}(p)}{\delta \mu^+(p)} = \frac{\mu(p) \left[ 1 - \frac{p^2 \bar{c}_0(p)}{\mu \mu_0 (1 + \frac{\bar{c}_0}{\mu_0} \mu)} \right]}{2(p^2 + \mu \mu^+)} \quad (\text{A2.28})$$

One can see, that  $d(p) \neq d^+(p)$ , and the extra term in (A2.27), as compared to (A2.28), contains additional factor  $\frac{1}{\mu_0^2}$ .

To the lowest order in  $c_0$  ( $c_0 \sim \left(\frac{\rho}{R}\right)^4$ ), one has

$$d(p) \cong \frac{u_0^2(p)}{2\mu_0^2} \int \frac{d^4 p'}{(2\pi)^4} \mu(p') \bar{c}_0(p') + \frac{\mu^+(p)}{2(p^2 + \mu \mu^+)} \quad (\text{A2.29})$$

$$d^+(p) = \frac{1}{2} \frac{\mu(p)}{p^2 + \mu\mu^+(p)}. \quad (\text{A2.30})$$

One can see, that the small values of  $\mu_0$  and  $\mu(p)$  can occur as solutions of (A2.16) due to the first term in (A2.29).

Note, that the first term on the r.h.s. of (A2.29) corresponds to the right zero mode  $\sim \frac{1+\gamma_5}{2}$ , and  $\mu_0$  in the denominator of this term contains  $\mu(p)$ . In the opposite case, when  $c_0(p) \sim \frac{1-\gamma_5}{2}$ ,  $\mu_0$  is proportional to  $\mu^+(p)$  and, consequently,  $d(p)$  and  $d^+(p)$  in (A2.28), (A2.29) interchange their places.

### Appendix 3

#### Derivation of equations (33) for the mass matrix

In the case of one flavor (or diagonal flavor matrix) the terms  $\varphi_{LR} = \varphi_1^+ = id^+, \varphi_{RL} \equiv \varphi_1 = id$  were found in Appendix 2 and can be written as (in case of no zero modes)

$$d(p) = \frac{\mu^+(p)}{2(p^2 + \mu\mu^+)}, \quad d^+(p) = \frac{1}{2} \frac{\mu(p)}{(p^2 + \mu\mu^+)} \quad (\text{A3.1})$$

and more general form see in (A2.29), (A2.30).

We now consider the case, when  $\{\mu_i\}, \{\mu_i^+\}$ , when  $\{\varphi_i\}, \{\varphi_i^+\}, i = 1, 2, 3, 4$  form  $SU(2)$  matrices in the flavor space. In this case

$$G = \frac{1}{\partial^2 + m^2} = \frac{1}{\hat{a} + \hat{b}\gamma_5}, \quad \hat{a} = \partial^2 + \frac{\mu^2 + \mu^{+2}}{2}, \quad \hat{b} = \frac{\mu^2 - \mu^{+2}}{2} \quad (\text{A3.2})$$

and the dash signs over  $\hat{a}, \hat{b}$  denote matrices in flavor space. In case, when  $\mu$  and  $\mu^+$  are matrices with equal elements  $b \equiv 0$  and one returns to expressions (A3.1). However, when  $\mu \neq \mu^+$ ,  $G$  can be written as

$$G = (1 - \gamma_5 \hat{a}^{-1} \hat{b}) \frac{1}{1 - (\hat{a}^{-1} \hat{b})^2} \hat{a}^{-1}, \quad (\text{A3.3})$$

and the pole structure of  $G$  is more complicated.

In (35) we have used (A3.1), in case when valid for  $\mu = \mu^+$ , and in this case  $K_{ik}^+ = \bar{K}_{ik}^+ = 0$ , and  $\varphi_i = \varphi_i^+ = \frac{\mu_i(p)}{2(p^2 + \mu^2)}$ .

In this case  $\hat{m}^2 \equiv \hat{\mu}^2 = \begin{pmatrix} \mu_1^2 + \mu_3\mu_4, & (\mu_1 + \mu_2)\mu_3 \\ (\mu_1 + \mu_2)\mu_4, & \mu_2^2 + \mu_3\mu_4 \end{pmatrix}$ . Inserting this expression into (29), one reproduces the system of equations (35).

## Appendix 4

We calculate here the term  $tr(G\Omega G\Omega)$ , which we can write as follows:

$$-trG\Omega G\Omega = tr(\hat{\partial}\hat{V} + \hat{V}\hat{\partial} + i\hat{V}^2)G(\hat{\partial}\hat{V} + \hat{V}\hat{\partial}G + i\hat{V}^2). \quad (\text{A4.1})$$

Writing explicitly in the coordinate space, one has

$$(\hat{\partial}\hat{V} + \hat{V}\hat{\partial})G = \int \gamma_\mu \gamma_\nu \left( \frac{\partial}{\partial x_\mu} V_\nu(x, y) + V_\mu(x, y) \frac{\partial}{\partial y_\nu} \right) G(y, z) d^4 y. \quad (\text{A4.2})$$

In the last term on the r.h.s. of (A4.2), one can differentiate by parts yielding

$$V_\mu(x, y) \frac{\partial}{\partial y_\nu} G(y, z) dy = \int \frac{\partial}{\partial y_\nu} (V_\mu(x, y) G(y, z)) d^4 y - \int \left( \frac{\partial}{\partial u_\nu} V_\mu(x, y) \right) G(y, z) dy. \quad (\text{A4.3})$$

Here the first term vanishes, and the second yields the minus sign, which allows to rewrite

$$(\hat{\partial}\hat{V} + \hat{V}\hat{\partial})G \rightarrow (\hat{\partial}\hat{V} - \hat{V}\hat{\partial})G \quad (\text{A4.4})$$

and for the total answer of terms with derivatives one obtains

$$-trG\Omega G\Omega = 4(\partial_\mu V_\nu)^2 - 4\partial_\mu V_\nu \partial_\nu V_\mu + \dots \quad (\text{A4.5})$$

where  $V_\mu \equiv V_{1\mu}^A t^A + V_{2\mu} y(a)$ , so that finally one has

$$-trG\Omega G\Omega = ((F_{\mu\nu}^A)^2 + 2F_{\mu\nu}^2 \sum_{a=1}^4 y^2(a))G^2 + \left( \frac{1}{2}(V_{1\mu}^A)^2 + V_{2\mu}^2 y^2 \right) G^2, \quad (\text{A4.6})$$

where

$$F_{\mu\nu}^A = \partial_\mu V_{1\nu}^A - \partial_\nu V_{1\mu}^A + e_{ABC} V_{1\mu}^B V_{1\nu}^C, \quad F_{\mu\nu} = \partial_\mu V_{2\nu} - \partial_\nu V_{2\mu}$$

and  $G_u G_d \approx G_u^2 \approx G_d^2 \equiv G^2$ , and

$$\begin{aligned} \text{tr}(G\hat{\Omega}G\hat{\Omega}) &= -(F_{\mu\nu}^A)^2 G_u G_d - F_{\mu\nu}^2 2 \sum_a y^2(a) + \\ &+ \left( \frac{1}{2} (V_\mu^A)^2 + V_{2\mu}^2 y^2 \right)^2 G^2 + \dots, \end{aligned} \quad (\text{A4.7})$$

which enters in the renormalization factor  $Z_W$ ,  $\sqrt{Z_W} V_{1\mu}^A = \tilde{V}_{1\mu}^A$

$$\frac{N_2}{4} (F_{\mu\nu}^A)^2 G^2 \equiv \frac{Z_W}{4} (F_{\mu\nu}^A)^2 = \frac{1}{4} (\tilde{F}_{\mu\nu}^A)^2 \quad (\text{A4.8})$$

and the same for  $F_{\mu\nu}, V_{2\mu}$ . Note, that after renormalization the renormalized  $(V)^4$  term enters with coefficient  $1/Z$ ,  $(V)^4 = \frac{\tilde{V}^4}{Z}$ , where  $Z \simeq \frac{N_2}{16\pi^2} \ln \frac{M^2}{\mu^2}$  and hence for large  $M$  this term can be neglected,  $N_2 = 4$  for  $SU(4)$  group.

The calculations done above in agreement with [14, 15] support the idea, that the quark loop expansion yields an effective Lagrangian for the composite vector field, where the gauge invariance is restored (seen e.g. in appearance of  $F_{\mu\nu}^A$  and  $F_{\mu\nu}$  in (A4.7)), up to the mass terms.

Let us now return to our case, where in addition to the term  $V_L^A t^A$  there is also  $V_R^A t^A$  and diagonal in  $SU(2)$  indices terms  $V_L$  and  $V_R$ . The EWSB combination is now

$$\text{tr}(\hat{G}\hat{N}\hat{G}\hat{N}) = \text{tr}\{(\bar{V}_L\hat{\mu} - \hat{\mu}\bar{V}_R)\hat{G}(\bar{V}_L\hat{\mu} - \hat{\mu}\bar{V}_R)\hat{G}\}. \quad (\text{A4.9})$$

To simplify matter, one can keep first only  $\hat{V}_L$ , then the general structure of (A4.9) comes out to be

$$(42) = \bar{c}_1 (V_L^A)^2 + (\bar{c}_3 V_L^A n^A + \bar{c}_2 V_L)^2 + \bar{c}_4 (V_L^A n^A)^2 \quad (\text{A4.10})$$

where  $\bar{c}_i$  depend on  $\mu_i$  and  $\hat{G}$ . One can see in (A4.10) explicitly the EWSB, which manifests itself both in the mixing between terms  $V_L^A$  and  $V_L$ , and in the last term on the r.h.s. of (A4.10). Note, that the vector  $n^A$  defines the “direction of the diagonalization” of the matrix  $\hat{\mu}$ ,  $\hat{\mu} = (a + \mathbf{n}\boldsymbol{\tau})\mu_d(a^* - i\mathbf{n}\boldsymbol{\tau})$ , which is the  $3d$  axis in the case of Standard Model. Thus one can visualize the effects of EWSB, however, one also obtains extra terms (e.g. the last term in (A4.10)) which should be missing in the realistic case. In the case, when also  $V_R^a$  and  $V_R$  are present, one can have spontaneous violation of left-right symmetry, however the analysis in the general case, when both  $\bar{V}_l$  and  $\bar{V}_R$  are nonzero and  $\mu \neq \mu^+$  becomes rather complicated and will be reported elsewhere [26].

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